Statistics

Summer 2023

Lecture 8

Feb 19-8:47 AM

Class QZ 8

Given $\boxed{P(A)=.4}$, $\boxed{P(B)=.8}$, $\boxed{P(A \text{ and } B)=.3}$

Find

1) $P(A \text{ or } B) = .4 + .8 - .3 = \boxed{.9}$ ✓

$= P(A) + P(B) - P(A \text{ and } B)$

De Morgan's Law

2) $P(\bar{A} \text{ and } \bar{B})$

$= P(\overline{A \text{ or } B}) = 1 - .9 = \boxed{.1}$ ✓

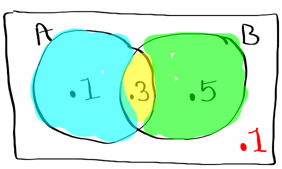
$= 1 - P(A \text{ or } B)$

3) $P(\bar{A} \text{ or } \bar{B})$

$= P(\overline{A \text{ and } B})$

$= 1 - P(A \text{ and } B)$

$= 1 - .3 = \boxed{.7}$ ✓



Total = 1

$P(A \text{ only}) = P(A) - P(A \text{ and } B)$

$= .4 - .3 = .1$

$P(B \text{ only}) = P(B) - P(A \text{ and } B)$

$= .8 - .3 = .5$

$P(A \text{ only OR } B \text{ only}) = .1 + .5 = \boxed{.6}$

Jun 22-11:42 AM

There are 3 women and 7 men.

We need to select 2 people.

Sample Space → WW WM MW MM

$P(WW) = \frac{3}{10} \cdot \frac{2}{9} = \frac{6}{90}$ $P(MW) = \frac{7}{10} \cdot \frac{3}{9} = \boxed{\frac{21}{90}}$

$P(WM) = \frac{3}{10} \cdot \frac{7}{9} = \boxed{\frac{21}{90}}$ $P(MM) = \frac{7}{10} \cdot \frac{6}{9} = \frac{42}{90}$

$P(2W) = \frac{6}{90}$ $P(1W\ 1M) = \frac{42}{90}$ $P(\text{No } W) = \frac{42}{90}$

#W	P(#W)
2	6/90
1	42/90
0	42/90

#W → L1, P(#W) → L2

Use $\boxed{\text{1-Var Stats}}$ with L1 & L2

To find $\bar{x} = .6$

$S = S_x =$ blank

$n = 1$

WW

$\boxed{\text{Some W, Some M}}$

MM

$P(\text{at least } 1W) = 1 - P(\text{No } W)$

these answers must be reduced.

$= 1 - P(MM) = 1 - \frac{42}{90} = \frac{48}{90}$

$P(\text{at least } 1M) = 1 - P(\text{No } M)$

$= 1 - P(WW) = 1 - \frac{6}{90} = \frac{84}{90}$

Jun 26-7:38 AM

A box 3 Quarters and 5 nickels.

Select 3 coins no replacement.

QQQ

$\boxed{\text{Some Q, Some N}}$

NNN

$P(QQQ) = \frac{3}{8} \cdot \frac{2}{7} \cdot \frac{1}{6} = \frac{1}{56}$

$P(NNN) = \frac{5}{8} \cdot \frac{4}{7} \cdot \frac{3}{6} = \frac{5}{28}$

$P(\text{at least } 1Q) = 1 - P(\text{No } Q) = 1 - P(NNN) = 1 - \frac{5}{28} = \boxed{\frac{23}{28}}$

$P(\text{at least } 1N) = 1 - P(\text{No } N) = 1 - P(QQQ) = 1 - \frac{1}{56} = \boxed{\frac{55}{56}}$

Sample Space

QQQ → 75¢

✓QQN → 55¢

✓QNQ → 55¢

QNN → 35¢✓

✓NQQ → 55¢

NQN → 35¢✓

NNQ → 35¢✓

NNN → 15¢

$P(75¢) = P(QQQ) = \frac{1}{56}$

$P(55¢) = 3 \cdot P(QQN)$

$= 3 \cdot \frac{3}{8} \cdot \frac{2}{7} \cdot \frac{5}{6} = \frac{15}{56}$

$P(35¢) = 3 \cdot P(QNN)$

$= 3 \cdot \frac{3}{8} \cdot \frac{5}{7} \cdot \frac{4}{6} = \frac{15}{28}$

$P(15¢) = P(NNN) = \frac{5}{28}$

Jun 26-7:49 AM

Total ¢	P(Total ¢)
75	1/56
55	15/56
35	15/28
15	5/28

Total ¢ → L1

P(Total ¢) → L2

Use $\boxed{\text{1-Var Stats}}$ with L1 & L2 to find

$\bar{x} = 37.5$

S = Blank

n = 1

P(at least 1 Q) =

1 − P(No Q) =

$1 - P(\text{All N}) = 1 - \frac{5}{28} = \boxed{\frac{23}{28}}$

P(at least 1 N) =

1 − P(No N) =

1 − P(All Q) =

$1 - \frac{1}{56} = \boxed{\frac{55}{56}}$

Jun 26-8:00 AM

L1	L2
1	.1
2	.15
3	.25
4	.35
5	.15

Use $\boxed{\text{1-Var Stats}}$ with L1 & L2 to find

$\bar{x} = 3.3$

S = Blank

n = 1

P(x = 2) = .25 P(x = 3) = .35 P(x = 4) = .2

P(x = 5) = .15 P(x = 6) = .05

x	P(x)
2	.25
3	.35
4	.2
5	.15
6	.05

x → L1 , P(x) → L2

Use $\boxed{\text{1-Var Stats}}$ with L1 & L2 to find

$\bar{x} = 3.4$

S = Blank

n = 1

$P(x \geq 3) = 1 - .25 = \boxed{.75}$

↑ Total Prob.

$P(x \leq 5) = 1 - .05 = \boxed{.95}$

Total Prob. ↓

Jun 26-8:08 AM

Suppose $P(A) = .4$, $P(B) = .3$,

A & B are $\boxed{\text{independent events}}$

1) $P(\bar{A}) = 1 - .4 = \boxed{.6}$

2) $P(\bar{B}) = 1 - .3 = \boxed{.7}$

3) $P(A \text{ and } B) =$

$P(A) \cdot P(B) = (.4)(.3) = \boxed{.12}$

4) $P(A \text{ or } B) =$

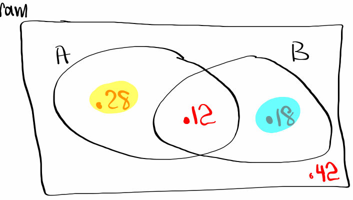
$P(A) + P(B) - P(A \text{ and } B) =$

$.4 + .3 - .12 = \boxed{.58}$

5) Make Venn Diagram

$.4 - .12 = .28$

$.3 - .12 = .18$



Total = 1

6) $P(\bar{A} \text{ or } \bar{B})$

$= P(\overline{A \text{ and } B})$

$= 1 - .12 = \boxed{.88}$

7) $P(\bar{A} \text{ and } \bar{B})$

$= P(\overline{A \text{ or } B})$

$= 1 - .58 = \boxed{.42}$

8) $P(A \text{ only or } B \text{ only}) = .28 + .18 = \boxed{.46}$

Jun 26-8:20 AM

5 Females, 10 Males, Select 3 people

1) How many ways can we select 3 people?

$_{15}C_3 = 455$

2) How many ways can we select 1 Female & 2 Males?

$_5C_1 \cdot {}_{10}C_2 = 225$

3) $P(1F \& 2M) = \frac{_5C_1 \cdot {}_{10}C_2}{_{15}C_3} = \frac{225}{455} = \boxed{\frac{45}{91}}$

4) $P(2F \& 1M) = \frac{_5C_2 \cdot {}_{10}C_1}{_{15}C_3} = \frac{100}{455} = \boxed{\frac{20}{91}}$

5) $P(\text{All Females}) = \frac{_5C_3 \cdot {}_{10}C_0}{_{15}C_3} = \frac{10}{455} = \boxed{\frac{2}{91}}$

6) $P(\text{All Males}) = \frac{_5C_0 \cdot {}_{10}C_3}{_{15}C_3} = \frac{120}{455} = \boxed{\frac{24}{91}}$

7) $P(\text{at least 1 Female}) = 1 - P(\text{No Females})$

$= 1 - P(\text{All males}) = 1 - \frac{24}{91} = \boxed{\frac{67}{91}}$

8) $P(\text{at least 1 Male}) = 1 - P(\text{No males})$

$= 1 - P(\text{All Females}) = 1 - \frac{2}{91} = \boxed{\frac{89}{91}}$

Jun 26-8:29 AM

Conditional Probabilities

SG 14

Multiplication Rule

$P(A \text{ and } B) = P(A) \cdot P(B | A)$

Given

with some algebra

$$\boxed{P(B|A) = \frac{P(A \text{ and } B)}{P(A)}}$$

$P(A) = .4$ $P(B) = .5$ $P(A \text{ and } B) = .25$

$P(B|A) = \frac{P(A \text{ and } B)}{P(A)} = \frac{.25}{.4} = \boxed{.625}$

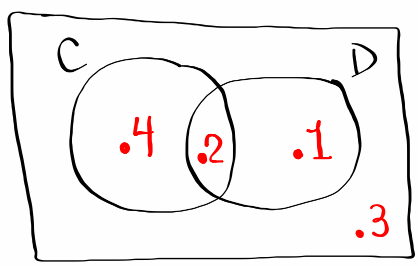
$P(A|B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{.25}{.5} = \boxed{.5}$

Jun 26-9:06 AM

$P(\text{Coffee}) = .6$

$P(\text{Donut}) = .3$

$P(\text{Coffee and Donut}) = .2$



Total = 1

$P(\text{Donut} | \text{Coffee}) = \frac{P(\text{Coffee and Donut})}{P(\text{Coffee})} = \frac{.2}{.6} = \frac{1}{3} = \boxed{.333}$

$P(\text{Coffee} | \text{Donut}) = \frac{P(\text{Coffee and Donut})}{P(\text{Donut})} = \frac{.2}{.3} = \frac{2}{3} = \boxed{.667}$

Jun 26-9:13 AM

$P(\text{iPhone}) = .8$

$P(\text{MAC}) = .3$

$P(\text{MAC} \mid \text{iPhone}) = .4$

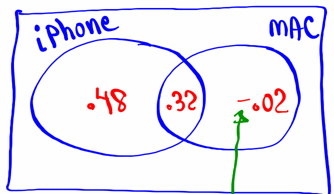
$P(\text{iPhone and MAC}) =$

$$P(\text{MAC} \mid \text{iPhone}) = \frac{P(\text{iphone and MAC})}{P(\text{iphone})}$$

$$.4 = \frac{P(\text{iPhone and MAC})}{.8}$$

Cross-Multiply

$P(\text{iPhone and MAC}) = .32$



You cannot have Neg. Prob.

Jun 26-9:19 AM

$P(\text{iPhone}) = .7$

$P(\text{MAC}) = .3$

$P(\text{MAC} \mid \text{iPhone}) = .4$

$P(\text{iPhone and MAC}) =$

$$P(\text{MAC} \mid \text{iPhone}) = \frac{P(\text{iphone and MAC})}{P(\text{iphone})}$$

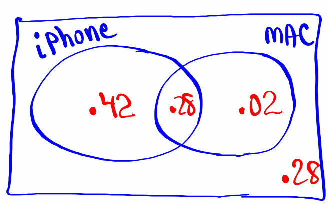
$$.4 = \frac{P(\text{iPhone and MAC})}{.7}$$

Cross-Multiply

$P(\text{iPhone and MAC}) = .28$

$.7 - .28 = .42$

$.3 - .28 = .02$



Total = 1

$P(\text{iPhone} \mid \text{MAC})$

$$= \frac{P(\text{iphone and MAC})}{P(\text{MAC})} = \frac{.28}{.3} = \boxed{.933}$$

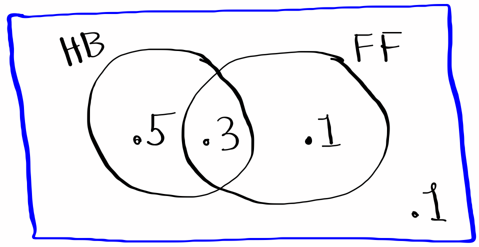
Jun 26-9:19 AM

$P(HB) = .8$

$P(FF) = .4$

$P(HB \text{ and } FF) = .3$

1) Make Venn Diagram



Total = 1

2) $P(FF \mid HB) = \dfrac{P(FF \text{ and } HB)}{P(HB)} = \dfrac{.3}{.8} = \boxed{.375}$

3) $P(HB \mid FF) = \dfrac{P(FF \text{ and } HB)}{P(FF)} = \dfrac{.3}{.4} = \boxed{.75}$

Jun 26-9:32 AM

3 Red, 4 Blue, 5 Green balls.

Select 3 balls, No replacement.

$P(\text{All Red}) = \frac{3}{12} \cdot \frac{2}{11} \cdot \frac{1}{10} = \boxed{\frac{1}{220}}$ ✓

$= \dfrac{{}_3C_3 \cdot {}_4C_0 \cdot {}_5C_0}{{}_{12}C_3} = \frac{1}{220}$ ✓

$P(\text{All Blue}) = \frac{4}{12} \cdot \frac{3}{11} \cdot \frac{2}{10} = \boxed{\frac{1}{55}}$ ✓

$= \dfrac{{}_3C_0 \cdot {}_4C_3 \cdot {}_5C_0}{{}_{12}C_3} = \frac{4}{220} = \boxed{\frac{1}{55}}$ ✓

$P(\text{All Green}) = \frac{5}{12} \cdot \frac{4}{11} \cdot \frac{3}{10} = \boxed{\frac{1}{22}}$ ✓

$= \dfrac{{}_3C_0 \cdot {}_4C_0 \cdot {}_5C_3}{{}_{12}C_3} = \frac{10}{220} = \boxed{\frac{1}{22}}$ ✓

$P(\text{One of each}) = \dfrac{{}_3C_1 \cdot {}_4C_1 \cdot {}_5C_1}{{}_{12}C_3} = \frac{60}{220} = \boxed{\frac{3}{11}}$

✓RBG ✓RGB
✓BRG ✓BGR
✓GRB ✓GBR

3 2 1

6 choices

$\overset{3}{\cancel{6}}\left(\frac{3}{12} \cdot \frac{4}{11} \cdot \frac{5}{\cancel{10}_{2}}\right) = \frac{3}{11}$ ✓

Jun 26-9:38 AM

P(at least 1 Red) = 1 − P(No Red)

$= 1 - \frac{{}_3C_0 \cdot \underbrace{{}_9C_3}_{\text{other Balls}}}{{}_{12}C_3}$

$= 1 - \frac{84}{220} = \boxed{\frac{34}{55}}$

P(at least 1 Blue) = 1 − P(No Blue) other Balls

$= 1 - \frac{{}_4C_0 \cdot {}_8C_3}{{}_{12}C_3} = 1 - \frac{56}{220} = \boxed{\frac{41}{55}}$

P(at least 1 Green) = 1 − P(No Green)

$= 1 - \frac{{}_5C_0 \cdot \overset{\text{other balls}}{{}_7C_3}}{{}_{12}C_3} = 1 - \frac{35}{220} = \boxed{\frac{37}{44}}$

Jun 26-9:54 AM

8 Females, 12 Males, Select 4 people,

$P(2F \,\&\, 2M) = \frac{{}_8C_2 \cdot {}_{12}C_2}{{}_{20}C_4} = \boxed{\frac{616}{1615}}$ ✓

$P(\text{at least 1 Female}) = 1 - \frac{{}_8C_0 \cdot {}_{12}C_4}{{}_{20}C_4} = \boxed{\frac{290}{323}}$ ✓

$P(\text{at least 1 Male}) = 1 - \frac{{}_8C_4 \cdot {}_{12}C_0}{{}_{20}C_4} = \boxed{\frac{955}{969}}$ ✓

SG 13 ✓

Jun 26-10:05 AM

SG 14

Data
1) Qualitative
2) Quantitative
- 1) Discrete "Countable"
- 2) Continuous "Measureable"

Jun 26-10:32 AM

Let x be a discrete Random Variable with Prob. distribution $P(x)$.

What is a prob. dist.?

It is a method that will provide the Prob. of all possible outcomes.

Prob. dist can be in the form of

1) Table
2) Graph
3) Formula

Jun 26-10:33 AM

Some Properties of prob. dist. P(x):

1) $0 \le P(x) \le 1$

2) $\sum P(x) = 1$

3) $P(x) = 0 \longleftrightarrow$ Impossible Event

4) $P(x) = 1 \longleftrightarrow$ Sure event

5) $0 < P(x) \le .05 \longleftrightarrow$ Rare event

Jun 26-10:37 AM

x	P(x)
1	.2
2	.5
3	.3

1) Verify $\sum P(x) = 1$. ✓

2) $P(x \ge 2) = 1 - P(x=1)$

$= 1 - .2 = \boxed{.8}$

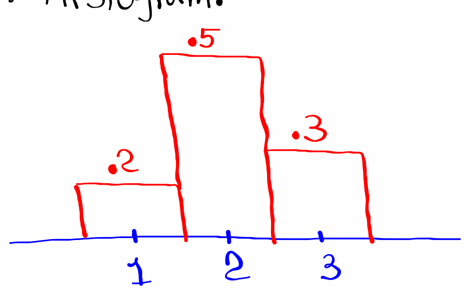
3) $P(x \le 2) = 1 - P(x=3)$

$= 1 - .3 = \boxed{.7}$

4) Draw Prob. dist. histogram.

$x \rightarrow$ Midpt

$P(x) \rightarrow$ Rel. F.



Jun 26-10:40 AM

Consider the chart below:

x	$P(x)$
1	.1
2	.3
3	.4
4	.2

1) $P(x=4) = 1 - \{.1 + .3 + .4\} = .2$

2) $P(x=1 \text{ or } x=4) =$

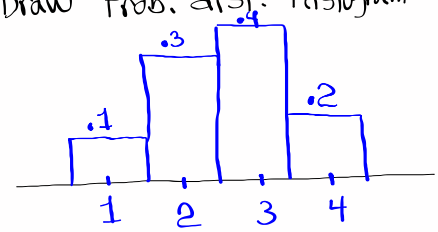
$.1 + .2 = \boxed{.3}$

3) $P(x>1) = P(x \geq 2)$

$= 1 - P(x=1)$

$= 1 - .1 = \boxed{.9}$

4) Draw Prob. dist. histogram



Jun 26-10:44 AM

Complete the chart below

x	$P(x)$	$xP(x)$	$x^2P(x)$
1	.2	.2	.2
2	.5	1.0	2.0
3	.3	.9	2.7

1) $\sum xp(x)$

$= .2 + 1.0 + .9 = \boxed{2.1}$

2) $\sum x^2 p(x)$

$= .2 + 2.0 + 2.7 = \boxed{4.9}$

3) $\sum x^2 p(x) - \left(\sum xp(x)\right)^2 = 4.9 - 2.1^2 = \boxed{.49}$

4) $\sqrt{\text{last answer}} = \sqrt{.49} = \boxed{.7}$

Jun 26-10:50 AM

Complete the Chart below

x	$P(x)$	$xP(x)$	$x^2P(x)$
1	.2	.2	.2
2	.3	.6	1.2
3	.4	1.2	3.6
4	.1	.4	1.6

1) $\sum P(x) = 1$ ✓

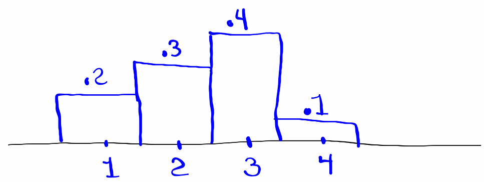
2) $\sum xp(x) = 2.4$

3) $\sum x^2 p(x) = 6.6$

4) $\sum x^2 p(x) - \left(\sum xp(x)\right)^2 = 6.6 - 2.4^2 = \boxed{.84}$

5) $\sqrt{\text{Last answer}} = \sqrt{.84} \approx \boxed{.917}$

6) Draw Prob. dist. histogram



Jun 26-10:56 AM

Mean μ "mu"

Variance σ^2 "Sigma^2"

Standard deviation σ "Sigma"

$$\boxed{\begin{array}{l} \mu = \sum x P(x) \\ \sigma^2 = \sum x^2 p(x) - \mu^2 \\ \sigma = \sqrt{\sigma^2} \end{array}}$$

x	$P(x)$	$xP(x)$	$x^2P(x)$
1	.3	.3	.3
2	.4	.8	1.6
3	.3	.9	2.7

$\mu = \sum x P(x) = .3 + .8 + .9 = \boxed{2}$

$\sigma^2 = \sum x^2 p(x) - \mu^2 = .3 + 1.6 + 2.7 - 2^2 = \boxed{.6}$

$\sigma = \sqrt{\sigma^2} = \sqrt{.6} = \boxed{.775}$

Jun 26-11:06 AM

How to find μ & σ using TI: List

$x \to L1$ use $\boxed{\text{1-Var Stats}}$ with $L1$ & $L2$

$P(x) \to L2$ FreqList

$\mu = \bar{x} = 2$

$\sigma = \sigma_x = .775$

$n = 1$ ✓

S_x blank ✓

What about σ^2? $.6$ ✓

$\boxed{\text{VARS}}$ $\boxed{\text{5: Statistics}}$ $\boxed{4{:}\ \sigma_x}$ $\boxed{x^2}$ $\boxed{\text{Enter}}$

Jun 26-11:13 AM

Given

x	$P(x)$
1	.15
2	.25
3	.35
4	.25

1) Verify $\sum P(x) = 1$ ✓

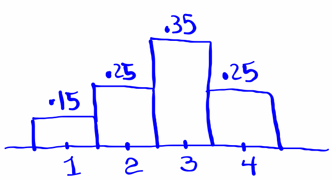
$x \to L1$, $P(x) \to L2$

Use $\boxed{\text{1-Var Stats}}$ with $L1$ & $L2$ to find

$\mu = \bar{x} = 2.7$ ✓S blank

$\sigma = \sigma_x = 1.005$ ✓$n = 1$

$\sigma^2 = \boxed{1.01}$



In Fraction

$\boxed{\text{MATH}}$ $\boxed{\text{1▸Frac}}$ $\boxed{\text{Enter}}$

$\frac{101}{100}$

$P(x \geq 2) = 1 - .15 = \boxed{.85}$

$P(x \leq 3) = 1 - .25 = \boxed{.75}$

Jun 26-11:18 AM

Class QZ 9

Answers in reduced fraction

4 Females, 6 Males, Select 3 people.

1) P(at least 1 Female) $= 1 - P(\text{All Males})$

$= 1 - \frac{6}{10} \cdot \frac{5}{9} \cdot \frac{4}{8} = \boxed{\frac{5}{6}}$

2) P(at least 1 Male) $= 1 - P(\text{All Females})$

$= 1 - \frac{4}{10} \cdot \frac{3}{9} \cdot \frac{2}{8} = \boxed{\frac{29}{30}}$

Jun 26-11:27 AM